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Absolute Instabilities in Stratified Shear Layers Created by Confinement

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Abstract

There is a fundamental question about the stability of stably stratified flows which arises in many different context such as the dynamics of the oceans, atmospheres or in industrial environmental processes. Our main motivation is to understand mechanisms of mixing.

A mixing layer is the region of high shear between two layers of uniform, but different, velocity. We investigate the absolute stability properties of such mixing layers. In temporal instability the disturbances are assumed to be periodic in the streamwise direction, and propagation properties are not determined. Absolute instability considers propagation of a spatially localized disturbance and determines whether there is a growth in the rest frame or not.

Huerre & Monkewitz 1985) [1] showed that mixing layers become locally absolutely unstable if there is a sufficiently strong reverse flow in one of the two streams, and then, disturbances spread and grow both upstream and downstream. Healey (2009) [2] showed that the presence of boundaries parallel to the shear layer can increase the absolute instability so that even mixing layers without reverse flow can become absolutely unstable. We show that for weakly stratified mixing layers typical of the upper ocean, the sea surface and the sea bed can provide the necessary confinement for the creation of local absolute instability.

Introduction

Consider Parallel stratified shear layers without reverse flow with bottom boundary varying slowly in streamwise direction. This is typical in the near shore region, river plumes, estuaries, etc. We consider the velocity profile and density profile shown in figure 1.

$$U(z) = (1 + \tanh(5z))/2 \text{ and } \bar{\rho}(z) = 1 - d\tanh(5z)\text{ , where } d = \frac{\rho_b - \rho_u}{\rho_b + \rho_u}. \tag{1}$$

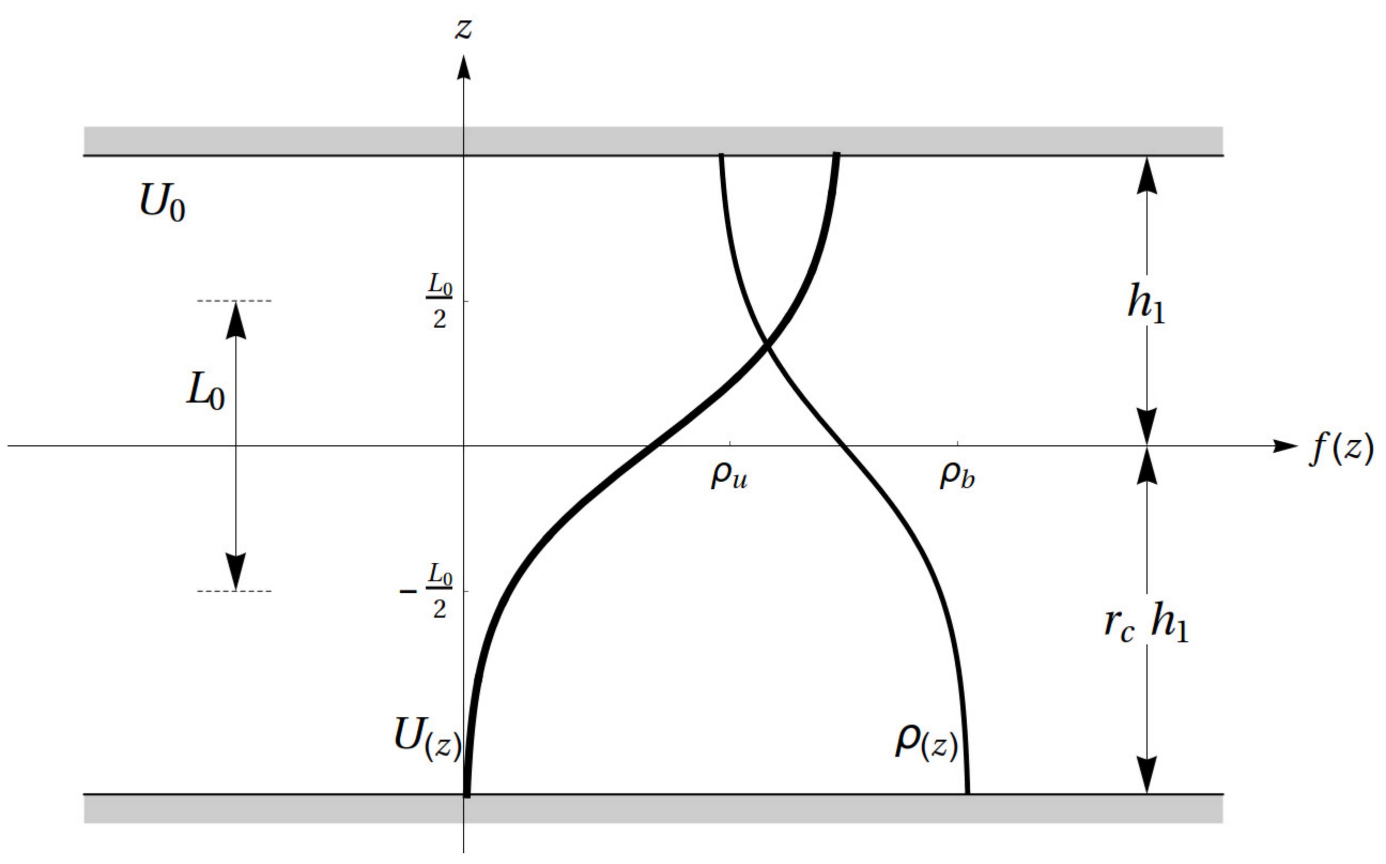

Figure 1: Schematic of the problem considered for temporal and absolute stability analysis with velocity profile and density profiles (1). h_1 and $r_c h_1$ are the distances of the upper boundary and bottom boundary respectively from the center of the velocity shear. r_c is the ratio of the upper and bottom boundary.

We non-dimensionalise velocity, length and density by U_0, L_0 and ρ_{mean}. We neglect viscosity, since we are investigating instabilities with relatively small time scales in comparison to viscous effect (large Reynolds number U_0L_0/ν). These flows are described by inviscid equations.

$$\left.\begin{aligned} \rho\left(\frac{\partial\bar{u}}{\partial t} + \bar{u}\cdot\nabla\bar{u}\right) &= -\nabla p + \bar{g}\rho \\ \bar{\nabla}\cdot\bar{u} &= 0 \\ \frac{\partial\rho}{\partial t} + \bar{u}\cdot\nabla\rho &= 0 \end{aligned}\right\} \tag{2}$$

We employ normal mode analysis

$$w = \hat{w}e^{i\alpha(x-ct)}, \qquad \text{with resultant frequency } \omega = c\alpha. \tag{3}$$

Equations of motion (2) can be linearised into Taylor-Goldstein [TG] equation

$$\underbrace{(U-c)\{D^2\hat{w} - \alpha^2\hat{w}\} - U''\hat{w}}_{\text{Rayleigh equation}} - \underbrace{\frac{\bar{\rho}'}{F^2(U-c)\bar{\rho}}\hat{w}}_{\text{Buoyancy term}} + \underbrace{\bar{\rho}'/\bar{\rho}\{(U-c)D\hat{w} - U'\hat{w}\}}_{\text{Inertia term}} = 0 \tag{4}$$

where $F = U_0/\sqrt{gL_0}$ is the Froude number and boundary conditions are (rigid lid conditions)

$$\hat{w} = 0 \text{ at } z = h_1, r_c\, h_1. \tag{5}$$

see Drazin and Reid 1981 [3]. The TG equation has been studied extensively for many years, but mostly for temporal instability. However temporal stability analysis will not tell us how a localised disturbance will behave and hence absolute stability analysis is needed.

Absolute instability

The space-time diagram in figure 2 indicates how disturbances can be convected as they grow.

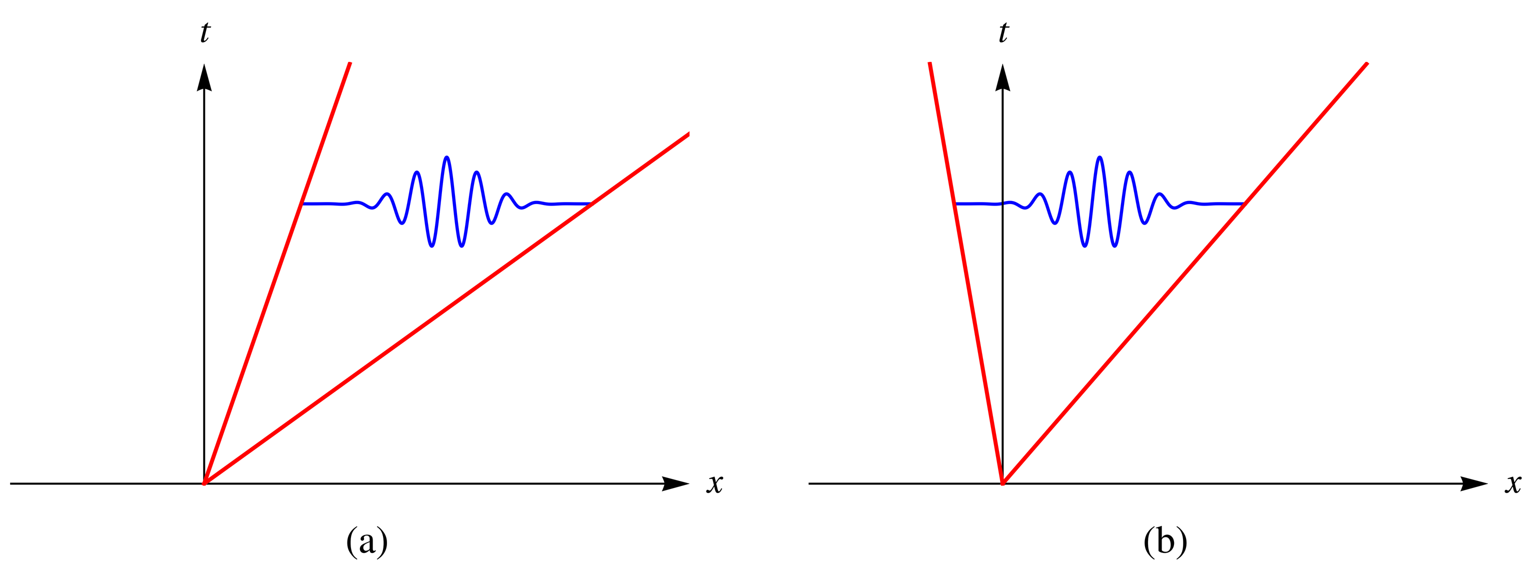

Figure 2: Results of local convective (a) and local absolute instability (b) in parallel shear layers. If the disturbance grows in time as it propagates away, eventually leaving the flow undisturbed in the frame of reference, the flow is locally convectively unstable (a). If the disturbance grows in time everywhere, eventually destroying the velocity profile in the frame of reference, the flow is locally absolutely unstable (b).

Absolute growth will mix the flow and destroy the initial profiles in a fixed frame of reference. To determine which case is important we use the Briggs saddle point method which requires for absolute instability

$$\frac{\partial\omega}{\partial\alpha} = 0, \qquad \mathrm{Im}(\omega) > 0, \tag{6}$$

where ω and α are complex quantities and hence results are in complex $\alpha-$plane, see Briggs 1964 [4] or Danyi 2018 [5] for details. In the original problems the boundaries are usually ignored and it is found that for absolute instability the reverse flow is necessary. Healey 2009 [2] showed that boundaries placed at particular places will produce absolute instability for no reverse flow. For example by adding top boundary at $h_1 = 5$ and bottom boundary at $h_2 = r_c\, h_1$, with $r_c = 2.5$. Adding boundaries to the problem results in poles along the α_i axis as shown in figure 3,

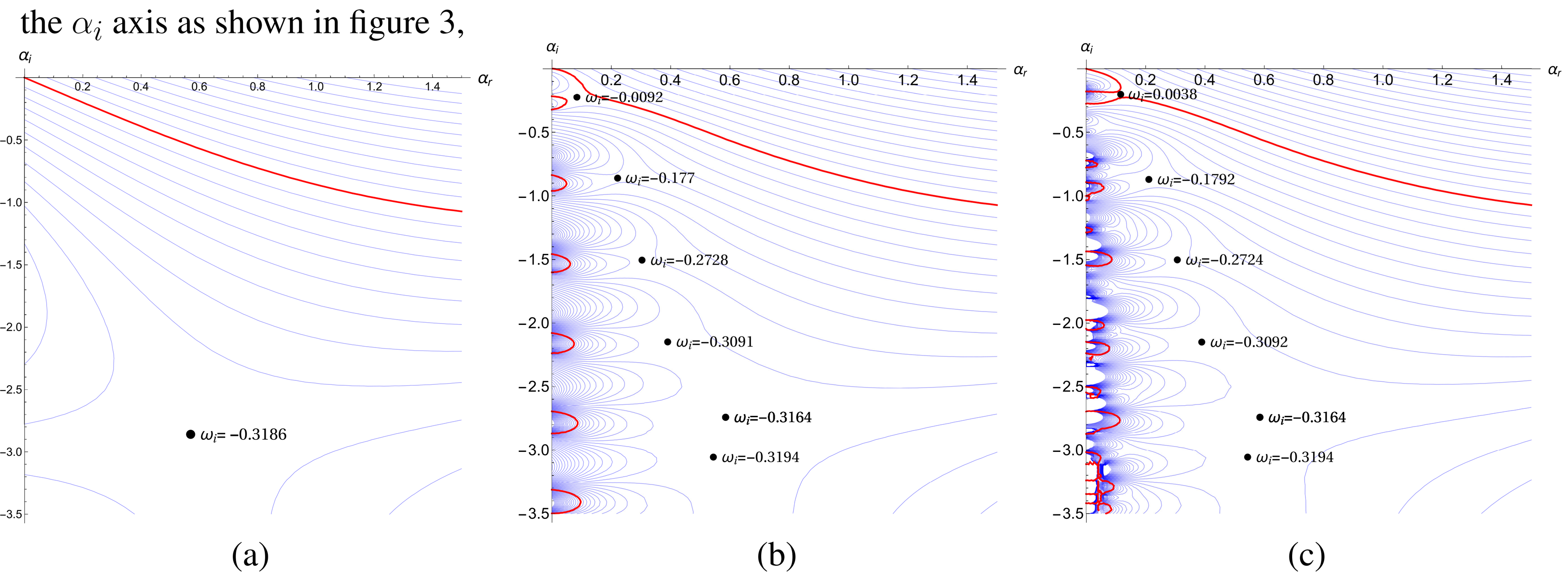

Figure 3: Absolute stability analysis results for unbounded case $h_1 = r_c = \infty$ (a), case with upper boundary only with $h_1 = 5, r_c = \infty$ (b) and case with upper and bottom boundary with $h_1 = 5, r_c = 2{,}5$ (c). Contours of constant Im(ω) (blue lines) in the complex α plane for solutions to TG (4). Saddles, where $\frac{\partial\omega}{\partial\alpha} = 0$, are marked as black discs with the dominant saddle being the one closest to the origin. The contours $\mathrm{Im}(\omega) = 0$ are indicated with a red line.

where in the case with both upper and bottom boundary added, figure 3(c), the saddle closest to the origin has positive value $\omega_i = 0.0028$ which corresponds to absolutely unstable mode.

We investigate the parameter space where absolute instability is present as shown in figure 4.

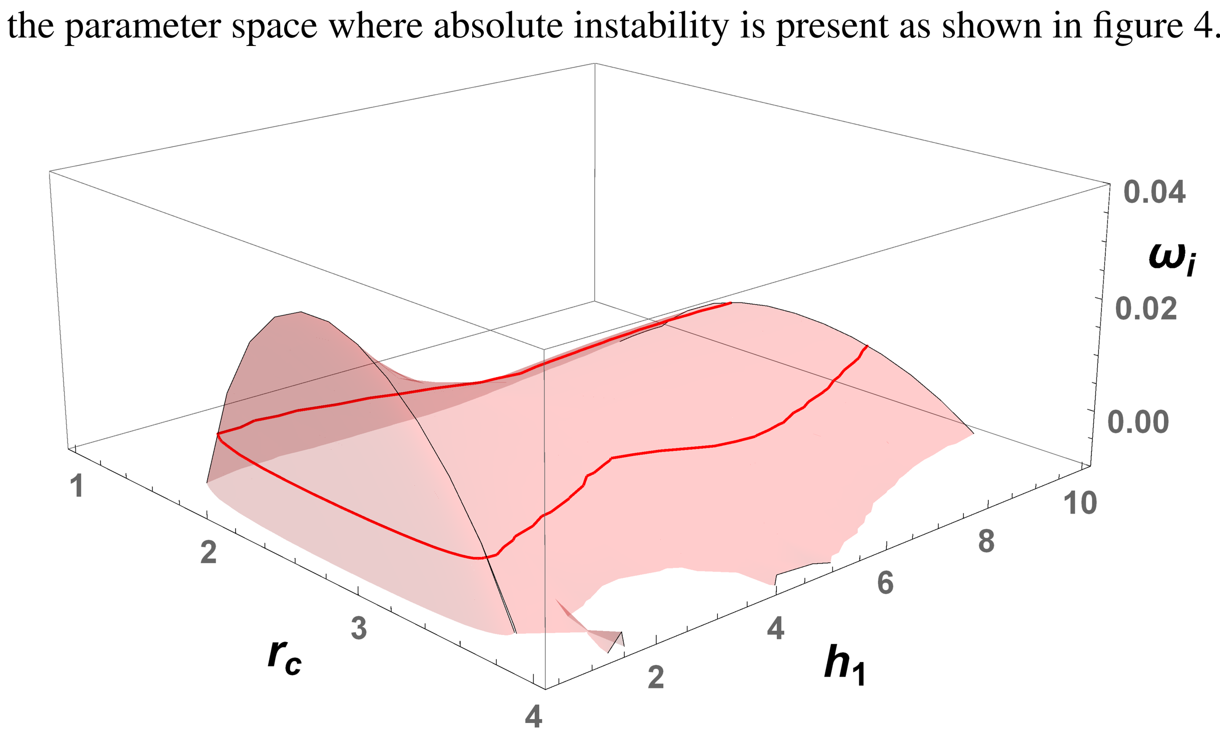

Figure 4: Absolute stability analysis results for homogeneous case. Surface of Im(ω) in parameter space of upper boundary h_1 and coefficient r_c for bottom boundary $r_c h_1$. The thick red line represents $\mathrm{Im}(\omega) = 0$ (neutral curve) and above which the flow is absolutely unstable.

Absolute instability is still present for stratified flows. To parametrise the stratification we define Richardson number $J_0 = dgL_0/U_0^2 = d/F^2$, where $d = (1 - \rho_u/\rho_b)/(1 + \rho_u/\rho_b)$ and ρ_u and ρ_b are upper and bottom densities of the flows.

Combination of neutral curves for varying J_0 leads to neutral surface in figure 5 showing parameter space where absolute instability is present. The maximum $J_0 \approx 0.025$ corresponds to boundaries placed at particular position with $h_1 \approx 1, r_c \approx 2.5$.

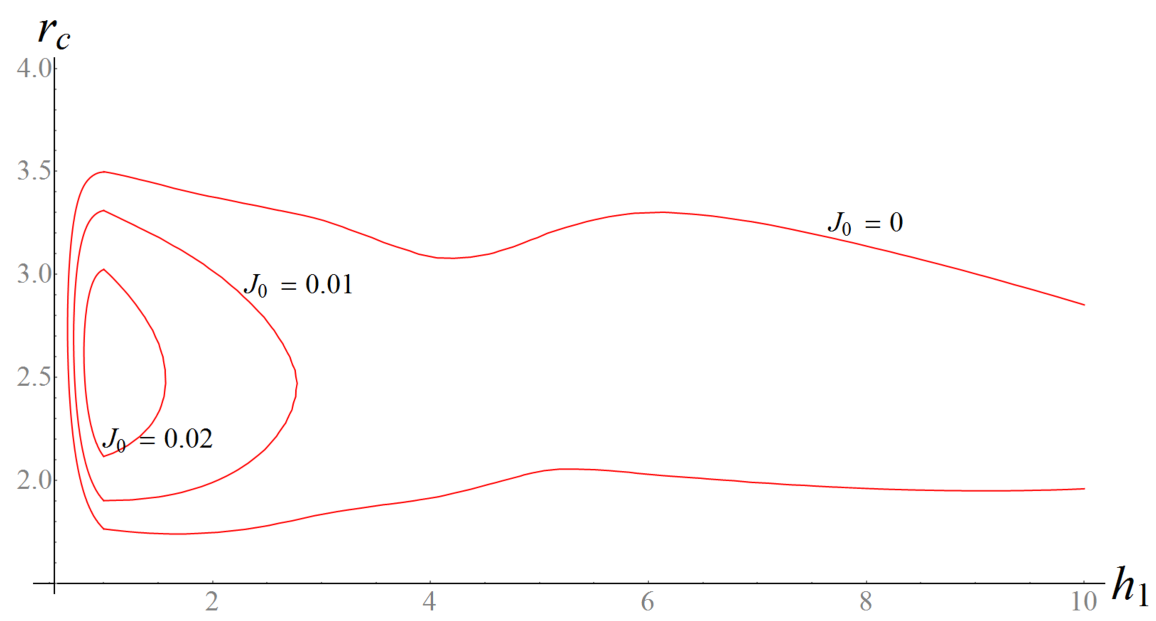

Figure 5: Absolute stability analysis results for stratified case. Contours of neutral surface where $\mathrm{Im}(\omega) = 0$ in parameter space of upper boundary h_1 and coefficient r_c for bottom boundary $r_c h_1$ for various values of J_0.

Conclusions

Following the work of Healey 2009 [2], where it was shown that boundaries added to an unbounded parallel flow can have (under certain parameter regime) destabilizing effect, we have extended this analysis to include the effects of stable stratification. Hence, we derive the parameter space for which the bounded co-flow with stable stratification is absolutely unstable. Under typical Mediterranean summer conditions the global Richardson number in the upper ocean is usually less than $J_0 = 0.025$. Hence in regions parallel to the shore, where the offshore (or inshore) wind creates a layer of fluid moving above essentially stationary fluid, sea bed topographies may result in regions where absolute instability is present.

References

[1] P. Huerre and P. Monkewitz, “Absolute and convective instabilities in free shear layers,” *J. Fluid Mech.*, vol. 159, pp. 151–168, 1985.

[2] J. Healey, “Destabilizing effects of confinement on homogeneous mixing layers,” *J. Fluid Mech.*, vol. 623, pp. 241–271, 2009.

[3] P. Drazin and W. Reid, *Hydrodynamic stability*. Cambridge University Press, 1981.

[4] R. Briggs, *Electron-Stream Interaction with Plasmas*. MIT Press, 1964.

[5] R. Danyi, “Global Instability of Mixing Layers Created by Confinement.,” *PhD Thesis.*, Keele University, 2018.